Development of distributed time-variant gain model for nonlinear hydrological systems

XIA Jun[1, 2], WANG Gangsheng[1], TAN Ge[1], YE Aizhong[2] & G. H. Huang[3]

1. Key Laboratory of Water Cycle & Related Land Surface Processes, Institute of Geographic Sciences and Natural Resources Research, Chinese Academy of Sciences, Beijing 100101, China;
2. State Key Laboratory of Water Resources & Hydropower Engineering Sciences, Wuhan University, Wuhan 430072, China;
3. Faculty of Engineering, University of Regina, 3737 Wascana Parkway, Regina, Sask., S4S 0A2, Canada

Correspondence should be addressed to Xia Jun(email: xiaj@igsnrr.ac.cn)

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Abstract In this paper, a rainfall-runoff modeling system is developed based on a nonlinear Volterra functional series and a hydrological conceptual modeling approach. Two models, i.e. the time-variant gain model (TVGM) and the distributed time-variant gain model (DTVGM) that are built on the platform of Digital Elevation Model (DEM), Remote Sensing (RS) and Unit Hydrological Process were proposed. The developed DTVGM model was applied to two cases in the Heihe River Basin that is located in the arid and semiarid region of northwestern China and the Chaobai River basin located in the semihumid region of northern China. The results indicate that, in addition to the classic dynamic differential approach to describe nonlinear processes in hydrological systems, it is possible to study such complex processes through the proposed systematic approach to identify prominent hydrological relations. The DTVGM, coupling the advantages of both nonlinear and distributed hydrological models, can simulate variant hydrological processes under different environment conditions. Satisfactory results were obtained in forecasting the time-space variations of hydrological processes and the relationships between land use/cover change and surface runoff variation.

Keywords: hydrological process, nonlinearity, time-variant gain model, distributed hydrological model.

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Hydrological science is a branch of the earth sciences. To study the complexities of hydrological processes and the associated environmental problems, a systematic approach is desired. For instance, Dooge published *Linear Theory of Hydrological System*[1] in 1973. Singh (1988) published *Hydrological Systems*[2], with its Chinese version[3] being translated by the Yellow River Conservancy Commission in 2000. Ge (1999) carried out systematic studies on the hydrological linear system theory, and produced *Modern Flood Forecasting Technologies*[4]. In addition to the linear systematic approach, many hydrologists have been exploring nonlinear ways for presenting hydrological systems since the 1960s[5–15]. This is due to the fact that the hydrological processes are generally too complicated to be expressed as linear systems, due mainly to the heterogeneousness of the relationships and interactions between hydrological processes and spatial characteristics. The nonlinearity is not only a fundamental science problem revealing complexities of earth system, but also a field for pioneering advanced hydrological theories and applied research.

The nonlinearity involves many concepts and conflicts, such as deterministic vs. stochastic, order vs. disorder, simple vs. complex, quantitative vs. qualitative, partial vs. integrated. At present, although the general theory of nonlinear hydrology has not been completely established, the related research has been in progress. For example, in the early 1960s, Minshall[16] studied the unit floods hydrographs variations of five different precipitation intensities based on the experiment in a watershed with an area of 1.093×10^5 m^2. Later on, Amorocho and Overton (1971) verified Minshall's conclusions by other nonlinear analysis approaches. Thus, Minshall's study became one of the powerful evidences to explain the nonlinear relationship between rainfall and runoff in nature watersheds[15] .

In order to study the intrinsic errors and applicable range of the unit hydrograph theory in the linear hydrological system, Amorocho (1963) defined a linearity function, $l(t)$, based on the nonlinear Volterra function series[5]:

$$l(t)=\frac{\int_{-\infty}^{t}h(t)x(t-t)\mathrm{d}t}{\sum_{n=1}^{\infty}\int_{-\infty}^{t}\cdots\int_{-\infty}^{t}h_n(t_1,\cdots,t_n)\prod_{j=1}^{n}x\left(t-t_j\right)\mathrm{d}t_1\cdots\mathrm{d}t_n},$$

where x is system input (e.g., rainfall); h_i is the ith-order response function; t, t_i (i=1,2,⋯, n) are time variants. According to the experiment with rainfall simulator, the inflow $x(t)$ is given in the forma of step function, rectangle function and the rectangle sequential function. The results indicated that there are significant nonlinear relations between rainfall and direct runoff in the hydrological process, except in floods recession periods. The hydrological process is by no means of linearity.

Singh (1975) made another experiment for the nonlinear kinematic wave model and the linear Nash model in an indoor hydrological laboratory with a rainfall simulator. Based on the analysis of results from 210 experiments, he pointed out that there existed high nonlinearity in surface runoff processes. Thus, the linear model was only an approximation to such processes; nonlinear model is desired. In 1980, hydrologists from the Institute of Geography Research, Chinese Academy of Sciences, carried out the artificial rainfall-runoff experiments under the uniform condition. They concluded that both the surface runoff processes and the rainfall runoff relations are nonlinear[17]. Since the 1980s, with the advances and application requirements on hydrological observation and information technologies, the system identification theory was applied to hydrological nonlinear modeling and parameterization. During this period, Xia develops the hydrological nonlinear identification approach to deal with difficulties of nonlinear hydrological modeling and parameter estimation[11,13−15,18]. The approaches include nonlinear analysis, system characterization, identification of analytic or quasi-analytic approaches for studying interactions among various system factors.

During the 1990s, Xia participated the International Workshop of River Flow Forecasting which was held by the University College of Galway (UCG), Ireland[11,13,18]. Through the joint research program, Xia1,2) analyzed hydrological data from forty different scale basins all over the world, and developed a simple time variant gain nonlinear model that can be transformed into the classic nonlinear hydrological model, i.e. the Time-Variant Gain Model (TVGM). The results showed that the TVGM's effectiveness in rainfall-runoff simulation was improved compared with that of linear models. By the end of the 20th century, the study of watershed hydrological modeling has been extended from lumped systems to distributed ones, due to the demand for considering the impacts of climate change and human activities[19, 20]. The development of distributed hydrological models requires more integration of hydrological and spatial information. Hydrological scale and nonlinear issues have be-

1) Xia Jun, Part 2: Nonlinear system approach, Research Report of the 3rd International Workshop on River Flow Forecasting, UCG, Ireland, 1989.
2) Xia Jun, Real-time rainfall-runoff forecasting by time variant gain models and updating approaches, Research Report of the 6th International Workshop on River Flow Forecasting, UCG, Ireland, 1995.

come hotspots in hydrological research.

The main characteristics of distributed hydrological modeling are: (1) the combination of hydrological physical model, conceptual model or systems model with Digital Elevation Model (DEM), GIS and RS technology will help extract important hydrological information such as slope degree, slope direction, transportation network, flow routing and watershed boundary; and (2) the integration of unit hydrological process with land use and cover change will help recognize hydrological response to change environment such as climate change and human activities. Basically, there are three distributed modeling approaches: (1) distributed physical models, such as MikeSHE[20–22], applying numerical methods based on rainfall-runoff models and DEMs; (2) distributed conceptual models, such as the distributed Xinanjiang model and Soil and Water Assessment Tool (SWAT)[23–25], integrating the existing lumped conceptual models with grids or sub-basin; and (3) semi-distributed models, such as TOPMODEL[20], simulating hydrological processes by applying topographic information.

The physically based rainfall-runoff models, such as MikeSHE, have the advantage based on physical theory in terms of hill slope hydrology. The problem of this approach, however, is its difficulties in terms of input data requirement, parameter interaction and scale issue from hillslope hydrological process to large basins. This led to the development of simplified distributed models such as the distributed Xinanjiang model and TOPMODEL. Such kind of distributed models are more flexible than fully distributed ones. However, there are still many assumptions associated with them, "the runoff generation can happen only when soil water is over the storage capacity of surface layer" as shown in the Xinanjiang model, as well as the lump input by averaged precipitation in the basin. Such distributed models are suitable only to humid regions. However, the models could hardly be applied to arid and semiarid regions such as northern and northwestern China. Therefore, it needs to develop a new distributed hydrological model to overcome the difficulties associated with the existing models.

A new challenge of the 21st century to hydrological modeling is enhancing our capability in the prediction of the hydrological process in limited data or ungauged basins, i.e., to build the hydrological model for simulating the changing environment with limited hydrological observation in developing countries. Currently, the International Association of Hydrological Sciences (IAHS) has recently launched a 10-year initiative, called the IAHS Decade for Prediction in Ungauged Basins (PUB), to address this problem[26]. The previous studies on basin hydrological modeling showed that the hydrological system approach, applied to basin hydrological modeling, seems to be more flexible than the physical or conceptual models under the conditions of data limit and uncertainty perturbation and environment change. Meanwhile, the distributed physical model approach could provide more physical process explanations and understandings. Thus, these are important issues for hydrologists in strengthening the combination of distributed physical model and hydrological nonlinear system theory, to simulate the nonlinear mechanism of hydrological system of change environment, and to set up a hybrid distributed hydrological model with theoretical and practical aspects.

This paper put forward a distributed hydrological model combining nonlinear system and physical models. Firstly, the general formulation of Volterra nonlinear system and the TVGM model are introduced, with their interrelations being highlighted. Then, the distributed time-variant gain model (DTVGM) coupled with unit hydrological model and GIS is developed. The developed approach is applied to two typical basins in China. The results indicate that performance of the DTVGM is reasonable. The integration of nonlinear theory and distributed hydrological model provides an effective means for simulating variable and complex environmental conditions.

1 Hydrological nonlinear system and time-variant gain model (TVGM)

For a lumped system, there are two approaches on nonlinear hydrological system theories and mathematic solutions. One is the integration formulation in terms of the Volterra functional series[2,5,11,14,27,28] that describes system behavior by input, output and system response functions. The general nonlinear system is given as follows:

$$y(t)=h_0+\int_{-\infty}^{t}h_1(t)x(t-t)\mathrm{d}t+\int_{-\infty}^{t}\int_{-\infty}^{t}h_2(t_1,t_2)\cdot x(t-t_1)x(t-t_2)\mathrm{d}t_1\mathrm{d}t_2+\cdots+\int_{-\infty}^{t}\cdots\int_{-\infty}^{t}h_n(t_1,\cdots,t_n)x(t-t_1)\cdots x(t-t_n)\mathrm{d}t_1\cdots\mathrm{d}t_n, \tag{1}$$

where x is system input (e.g., rainfall), y is output (e.g., runoff); h_i is the ith-order response function; t, t_i (i=1, 2,⋯, n) are time variants. In this equation, the response functions express the system function. For the rainfall-runoff system, the response functions mainly reflect the basic characteristics and system effects of watersheds.

In hydrological modeling, the general hydrological system is linear, and the second-order functional series is enough to express the nonlinear behavior of the rainfall-runoff process. Therefore, the nonlinear hydrological model in eq. (1) can be approximately expressed as

$$y(t)=\int_0^L h(t)x(t-t)\mathrm{d}t+\int_0^L\int_0^L g(t,s)x(t-t)x(t-s)\mathrm{d}t\mathrm{d}s, \tag{2}$$

where $h(t)$ is linear response function; $g(t, s)$ is nonlinear second-order response function; L is memory length; t, s are time variants.

The second nonlinear system formulation is the differential equations based on micro scale, including the descriptions of mass balance and dynamic process[29], given by

$$\begin{cases}\dfrac{\mathrm{d}S}{\mathrm{d}t}=x-y,\\ S=\sum\limits_{m=1}^{M}a_m(x,y)\dfrac{\mathrm{d}^{(m)}x}{\mathrm{d}t^{(m)}}+\sum\limits_{n=1}^{N}b_n(x,y)\dfrac{\mathrm{d}^{(m)}y}{\mathrm{d}t^{(m)}},\end{cases} \tag{3}$$

where S is the unit storage (m^3); x is the unit inflow (m^3/s); y is the unit outflow (m^3/s); $a_m\{\cdot\}$ are m-order nonlinear coefficients of inflow rate; $b_n\{\cdot\}$ are n-order nonlinear coefficients of outflow rate.

It should be pointed out that though the formulas in eq. (2) and eq. (3) are different, they could be transferred to each other under certain conditions. For example, assuming the behavior of basin system is equivalent to a cascade of nonlinear reservoirs, and the relationship between S and y can be expressed as the following nonlinear differential equations:

$$\begin{cases}\dfrac{\mathrm{d}S_i(t)}{\mathrm{d}t}=x_i(t)-y_i(t),\\ y_i(t)=aS_i^{\,m}(t),\end{cases} \tag{4}$$

where a is storage parameter; m is nonlinear storage parameter. Then, the nonlinear parameter system model with the formulation of Volterra functional series, correspondent to the nonlinear differential equations (4), could be developed[14,27], in which the linear and nonlinear response functions can be expressed as

$$\begin{cases}h(t)=\dfrac{(t/K)^{n-1}}{K(n-1)!}\mathrm{e}^{-t/K},\\ g(t,s)=b\left[h_n'(t)\sum\limits_{j=1}^{n}h_j'(s)+h_n'(s)\sum\limits_{j=1}^{n}h_j'(t)-h_n'\{\max(t,s)\}\right],\end{cases} \tag{5}$$

where $h_j'(t)=\dfrac{(t/K)^{j-1}}{(j-1)}\mathrm{e}^{-t/K}$, K is a storage parameter of Nash unit hydrograph model. For the conceptual model of eq. (5), the system characteristics depend on parameters (n, K, b).

Evidently, the nonlinear system formulation and its resolution are rather more complex than the linear system. It leads to a barrier to development and application of the hydrological nonlinear approach. Due to the complexity of hydrological system, many previous studies had to focus on the identification issues of nonlinear mathematic models related to eq. (1) or eq. (2), and tried to solve the difficulties of nonlinear modeling[6,8,9,11,18,30–32]. However, over a long period of time, the nonlinear hydrological theory has been confronted with a scientific challenge, i.e., is it possible to find out a simple system relation instead of the complex nonlinear expression by eq. (2)?

In the research period of International Workshop of River Flow Forecasting, held at the University College of Galway (UCG), Ireland during 1989–1995, Xia presented a Time Variant Gain Model (TVGM). One of major contributions is to develop a simple relationship between the time-variant runoff coefficient, $G(t)$, and soil moisture (or antecedent precipitation index, API), in terms of hydrological data set of more than forty basins in the world collected by UCG, given by

$$G(t) = g_1 \mathrm{API}^{g_2}(t), \tag{6}$$

where g_1 and g_2 are parameters related to $G(t)$; API(t) is the Antecedent Precipitation Index function (API), to be used as an index of the degree of catchment wetness, defined in the present context as the outflow from a linear reservoir having the total rainfall function, $X(t)$, as input, i.e.

$$\begin{aligned} \mathrm{API}(t) &= \int_0^t U_0(s)X(t-s)\mathrm{d}s \\ &= \int_0^t \frac{\exp\left(-\dfrac{s}{K_\mathrm{e}}\right)}{K_\mathrm{e}} X(t-s)\mathrm{d}s, \end{aligned} \tag{7}$$

where K_e is a parameter that indicates the rate of recession of soil moisture[12,13].

Using the system concept of time-variant runoff coefficient, a simple model TVGM with a few of parameters can be established, where the runoff generation process can be expressed as

$$R(t) = G(t)X(t), \tag{8}$$

where $X(t)$ is system input (e.g., rainfall); $Y(t)$ is output (e.g., runoff). Furthermore, the flow routing process of the basin system can be described by the unit hydrograph method, given by

$$Y(t) = \int_0^m U(t)R(t-t)\mathrm{d}t, \tag{9}$$

where $U(\hat{o})$ is response function. It can be proved that the simple model described from eq. (6) to eq. (9) is equivalent to the nonlinear Volterra model. For example, expanding eq. (6) as a Taylor series, its first two parts are

$$G(t) = a + b\,\mathrm{API}(t), \tag{10}$$

where a and b are coefficients. Substituting eq. (7) into eq. (10), and then substituting eq. (10) into eq. (8), the relationship between $R(t)$ and $X(t)$ could be obtained as

$$\begin{aligned} R(t) &= G(t)X(t) \\ &= (a + b\,API(t))X(t) \\ &= a\,X(t) + b\,API(t)X(t) \\ &= a\,X(t) + \int_0^t bU_0(s)X(t-s)X(t)\mathrm{d}s\,. \end{aligned} \tag{11}$$

Finally, substituting eq. (11) into eq. (9), an isomorphic representation of Volterra model can be deduced as

$$\begin{aligned} Y(t) &= \int_0^m U(t-t)R(t)\mathrm{d}t \\ &= \int_0^m aU(t-t)X(t)\mathrm{d}t \\ &\quad + \int_0^m bU(t-t)API(t)X(t)\mathrm{d}t \\ &= \int_0^m aU(t-t)X(t)\mathrm{d}t \\ &\quad + \int_0^m \int_0^t bU_0(t-s)U(t-t)X(s)X(t)\mathrm{d}t\,\mathrm{d}s \\ &= \int_0^m H_1(t-t)X(t)\mathrm{d}t \\ &\quad + \int_0^m \int_0^t H_2(t-s, t-t)X(s)X(t)\mathrm{d}t\,\mathrm{d}s, \end{aligned} \tag{12}$$

where $H_1(t–t)=aU(t–t)$, $H_2(t–s, t–t)=bU_0(t–s)U(t–t)$; t, s are time variants; m is memory length. The above nonlinear systems analysis indicates that the simple TVGM could be used to replace the complex Volterra nonlinear formulation, since there exists an intrinsic relation between them. The TVGM model can also be used for real-time forecasting for hydrological systems with nonlinear seasonal disturbances[13,14]. Verified with real data, the simulation accuracy of TVGM is significantly improved compared to those of other linear models (Table 1).

2 Distributed time-variant gain model

The Distributed Time-Variant Gain Model (DTV-

Table 1 Comparison of efficiency of TVGM model with TRLM model

Class	Catchment name	Area/km^2	Country	Parameter		Calibration efficiency R^2		Verification efficiency R^2	
				m	K_e	TRLM	TVGM	TRLM	TVGM
A	Bird Creek	2344	USA	10	20	59.24	87.67	–52.70	43.29
	Wolombi	1580	Australia	10	20	46.23	83.57	–13.37	36.70
B	Kizu	1445	Japan	10	20	80.00	87.96	67.41	75.17
	Chaiping	2370	China	40	40	56.68	75.12	61.63	77.37
	Xichuang	3092	China	10	20	71.95	87.54	53.91	74.75
C	Nam Mune	104000	Thailand	30	100	50.27	78.89	54.09	85.78
	Daqingjiang	61780	China	10	20	70.12	83.96	70.62	78.31
	Baihe	15300	China	10	20	79.28	84.34	82.96	88.60
	Yangxian	23805	China	10	20	72.44	87.11	61.26	80.04

A, Arid and semiarid catchment with inconsistent data series in model calibration and verification; B, semiarid or monsoon influenced mid-small catchment with consistent data set in model calibration and verification; C, monsoon influenced mid-large catchment with consistent data set in model calibration and verification; TRLM: Total Runoff Linear Model; TVGM: Time-Variant Gain Model.

GM) is an extension of the nonlinear approach for simulating distributed hydrological basin using GIS/RS platform and hydrological process information at a local scale. For instance, based on GIS/DEM information, it could pick up the information of the gradient of the slope, the direction of the slope, the flow direction, and the boundaries of the basin. By combining the hydrological processes modeling of evapotranspiration, soil water, and snowmelt with the distributed inputs, the concept and method of TVGM can be extended to modeling of surface runoff generation for a distributed system. Furthermore, the calculation of runoff generation in the sub-surface flow and ground flow are also completed in terms of water balance, storage relation and their integration in the distributed basin. Finally, the flow routing of a distributed basin can be simulated based on ranked grids (i.e., concentration belts) and input of runoff generation with spatial distribution information.

Basically, DTVGM includes multiple components of hydro-information analysis and modeling, such as distributed input data processing, runoff generation model on each grid unit and flow routing model between adjacent concentration belts[15,33]. Characteristics of DTVGM are given as follows: (1) It can describe time-space variation of rainfall, evapotranspiration and land cover based on DEM grid and spatial digit information; and (2) it combines runoff generation process and flow routing process together by soil moisture content, and carries out the hydrological simulation based on grid elements and stream network. Runoff generation occurs at each grid element with two or three layers in the vertical direction. Take the two-layer model divided by the ground surface for example, the upper layer is the surface runoff generation layer, while the lower one is the subsurface runoff generation layer. In the case study on the Heihe Mountainous basin, as a high-cold mountain region, its runoff comes from glacier and snow melting as well as rainfall. Due to the adaptability of DTVGM, its model structure can be adjusted according to the watershed characteristics. The scheme of DTVGM applied to the Heihe River basin is shown in Fig. 1.

DTVGM is a kind of distributed conceptual hydrological model, and has the advantage of system approach. It is available to the case of input information imperfection such as the lack of enough precipitation and evaporation observations for distributed modeling. And it has few parameters, which can be estimated in terms of system identification approach to reduce uncertainty. Major parts of DTVGM are introduced as follows:

(1) Specialization of land surface and hydrological information. The spatial distribution information of the watershed features and hydrometeorological variants can be obtained by spatial interpolation or data assimilation technologies. In this process, the evapotranspiration simulation refers to the calculations of potential evapotranspiration (E_p) and actual evapotranspiration (E_a). Several methods can be adopted to simulate potential evapotranspiration, such as Penman-Monteith formula, Priestley-Taylor formula and Har-

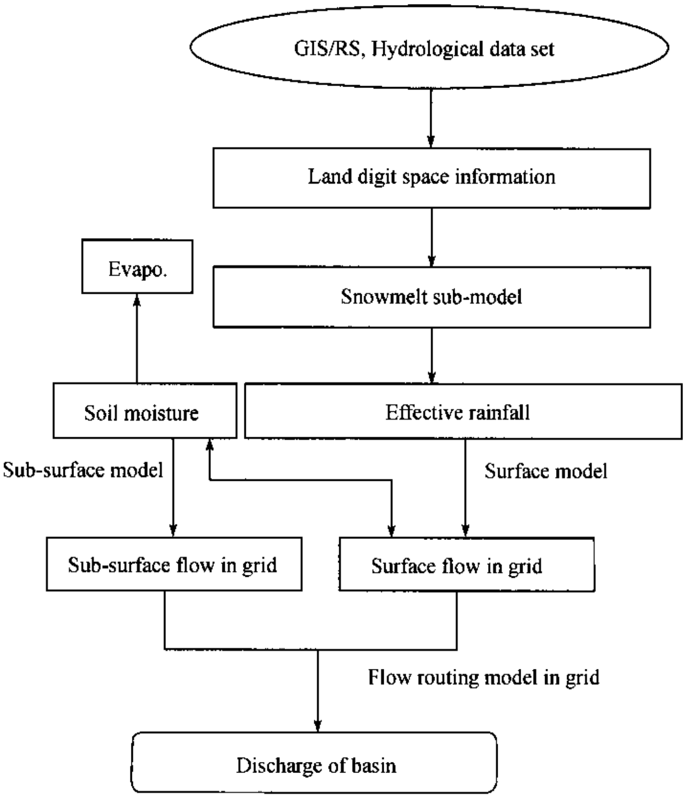

Fig. 1. Scheme of DTVGM applied to the Heihe River basin.

greaves method[24,34]. As to the actual evapotranspiration, it can be expressed as the function of Ep and soil moisture[35], i.e. Ea $=E$p · $f(S/W)$, where S is soil moisture, W is saturated soil moisture, and $f(S/W)$ is a linear or nonlinear function.

(2) Runoff generation simulation. According to the concept of TVGM, the surface runoff on each grid is calculated by $R_d = g_1 \cdot S^{g_2} \cdot P$, where R_d is surface runoff (mm); S is soil moisture (mm); P is precipitation (mm); g_1 and g_2 are undetermined coefficients. By introducing the non-dimensional variant S/W instead of S, the surface runoff of the ith grid at time t can be expressed as: $R_{dt,i} = g_1 \cdot (S_{t,i} / W_i)^{g_2} \cdot P_{t,j}$, where $R_{dt,i}$ is surface runoff (mm); $S_{t,i}$ is soil moisture (mm); W_i is saturated soil moisture (mm); $P_{t,i}$ is grid precipitation (mm). As for the subsurface runoff generation model, the subsurface runoff can be calculated by coupling the water balance equation and the dynamic storage-outflow function. The subsurface runoff of the ith grid at time t can be described as $R_{st,i} = K_g \cdot \frac{S_{t+1,i} + S_{t,i}}{2}$, where S_t and S_{t+1} are the soil moisture at time t and t+1; K_g is subsurface runoff generation coefficient. The total runoff on the ith grid $R_{t,i}$ is the sum of surface runoff and subsurface runoff, i.e. $R_{t,i}=R_{dt,i}+R_{st,i}$.

(3) Flow routing simulation. The flow routing model includes two aspects: the flow route and the routing method. In DTVGM, the flow route can be described as ranked grids (or concentration belts) according to the flow directions of grid elements. The outlet grids can be defined as the first rank, and the grids from which water flows into the first ranked grids are defined as the second rank, the rest may be deduced by analogy. Therefore, the most upstream grids are defined as the highest rank. Flow routing is undertaken from those grids having higher rank onto the grids having lower rank. The kinematic wave

routing[35], a more physically-based modeling approach, is adopted as the routing method in DTVGM. At present, the software system of DTVGM was developed to carry out case studies.

(4) Applications of DTVGM model. DTVGM model was applied to two basins in China. One is the Heihe mountainous basin in arid and semiarid region of Northwest China, and the other is the Chaobai River basin in semiarid and semi-humid regions of North China. The main assessment index chosen in this paper is the efficiency R^2[36]:

$$R^2 = (F_0 - F)/F_0,$$

where $F_0 = \sum_{k=1}^{N}[Y(k) - \bar{Y}]^2$, $F = \sum_{k=1}^{N}[Y(k) - \hat{Y}(k)]^2$; N is data length; $Y(k)$ is observed runoff; $\hat{Y}(k)$ is simulated runoff; $\bar{Y}$ is averaged observed runoff. Moreover, relative error of flow peak and the ratio of estimated average runoff to observed average runoff were also selected as an assessment index about system modeling and water balance relationship.

The Heihe mountainous basin, with an area of 10009 km^2, is the main runoff generation region of the Heihe River basin. The establishment of distributed hydrological model is meaningful for recognizing the evolution rules of water resources in the western inland region. It can also help to alleviate water use conflicts in the society-economy-ecology system, scientifically allocate water resources in the whole basin. The average elevation of the basin is 3670 m, ranging from 1737 to 5010 m. The DEM of the Heihe mountainous basin is divided into 38277 grids. The area of each grid (500 m × 500 m) is 250000 m^2. In addition, the grids of the basin are partitioned into 456 ranks (i.e. 456 concentration belts). As the basin belongs to a high-cold mountainous region, a sub-model of snowmelt process[15,33,37] was coupled into DTVGM. For the Heihe River basin with high mountain areas and semiarid regions, the hydrological and meteorological stations are insufficient, which makes it difficult for the hydrological modelling. Thus, the complex hydrological models are not suitable for such a basin, because the necessary data and parameters cannot be collected completely. However, the simple models like DTVGM are advantageous for dealing with the problem. Over six years daily hydrological data set from 1990 to1995 was selected for calibration of DTVGM (1990–1992) and verification (1993–1995). By comparison of the observed and estimated runoff (see Table 2 and Fig. 2), it was shown that the model's averaged efficiency of daily runoff simulation is over 0.75. The simulated stream-flow peaks, especially the larger discharges, have relatively high precision. Snowmelt runoff volume is 4.6% of the total runoff volume. Distributed modelling results are quite satisfactory.

The Chaobai River basin is located in the Haihe River basin, controlled by the Miyun reservoir with an area of 13846 km^2. It is the only surface water resource for Beijing City, the capital of China. Since the 1980s, the inflow to the reservoir was significantly decreased.

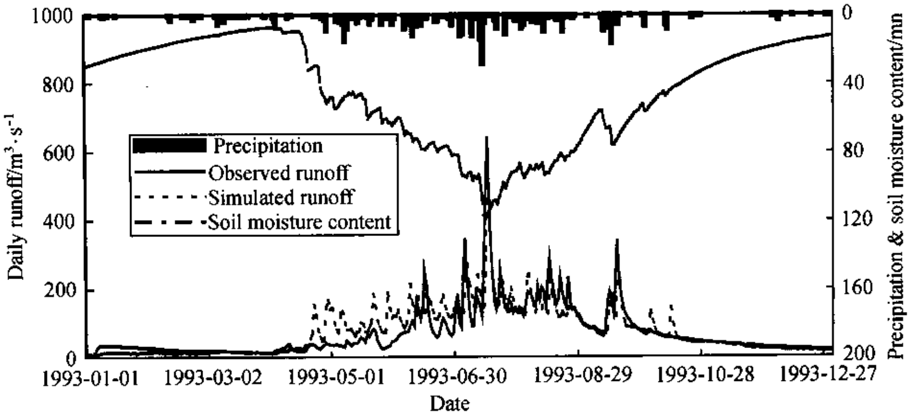

Fig. 2. Comparison between observed and simulated runoff in the Heihe River basin.

Table 2 Efficiency of DTVGM model applied to Heihe River basin

Year	Rainfall/mm	Observed yearly runoff/mm	Simulated yearly runoff/mm	Efficiency of modeling, R^2	Comparison of flow peak/$m^3 \cdot s^{-1}$		
					observed	simulated	relative error (%)
1990	346.4	158.23	166.23	0.80	485.0	479.5	1.1
1991	249.9	127.39	117.62	0.71	205.0	138.7	32.3
1992	332.0	132.09	150.01	0.79	233.0	264.5	13.5
1993	391.1	180.10	172.54	0.79	378.0	377.1	0.25
1994	278.6	140.39	137.81	0.73	242.0	225.2	6.9
1995	304.8	147.87	152.65	0.69	305.0	293.1	7.76

Therefore, it is essential to study the impacts of land use and cover change (LUCC) and human activities on water resources[15,38]. The elevation of the basin ranges from 130 to 2262 m. The whole basin is divided into 55444 grid elements with a mesh size of 500 m × 500 m. The results show that the simulation runoff volumes are similar to the observed ones, and the model efficiencies are above 0.8, which can satisfy the requirement of model application to water resources assessment and management.

Moreover, the relationship between LUCC and time variant gain factor, $G(t)$, was analyzed by using DTVGM model and RS information. The analysis processes are given as follows: (1) selected rainfall-runoff observed data set and calculate distributed time variant gain factor, G, in terms of DTVGM; and (2) using RS information (TM imaging information with 30 m × 30 m resolution) to identify its linkage of model gain factor, with different land cover types such as forest, grass and bear soil and so on. Evidently, for different land use/cover types, correspondent gain factor, G, is also different, which brings difficulties to build their direct relation. However, this relationship can be evaluated by stochastic approach, i.e. for a given threshold gain factor G_0, to analyze random distribution of accumulated frequency (DAF) of rate of a land cover area to its total area, PA ($G \geqslant G_0$). A result of the Chaobai River basin is shown in Fig. 4. It indicates that: (1) there are significant differences between any two various land use types, which means that LUCC will affect the runoff generation process; (2) in Fig. 4, dry land's DAF is the lowest, while that of forest and grass land is much higher, which means that during the arid period, the soil of dry land is much drier with the higher infiltration rate. As to forest and grass land, because of their grown root zones with higher soil water capacity, they are relatively easy to generate runoff compared to dry land; and (3) there is a change between forest's DAF and grass land's DAF. For this case, i.e., G_0<0.0028, runoff generation amount of forest cover is greater than that of grass land. However, for the case of G_0 >0.0028, the runoff

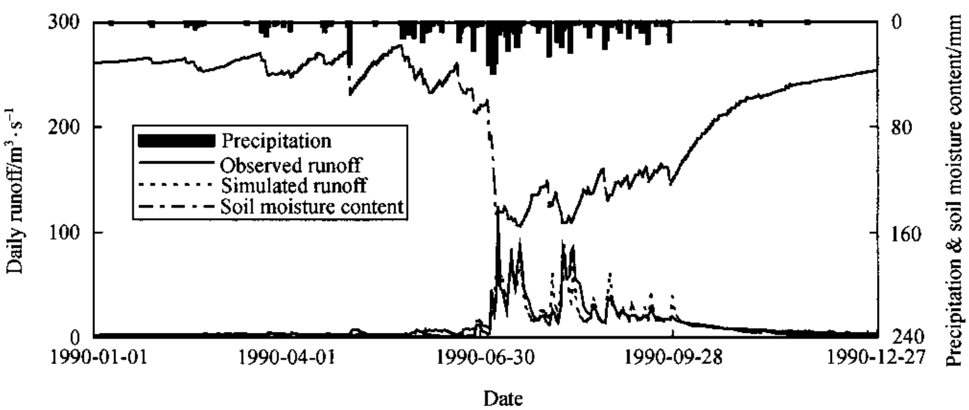

Fig. 3. Comparison between observed and simulated runoff of Chaobai River basin.

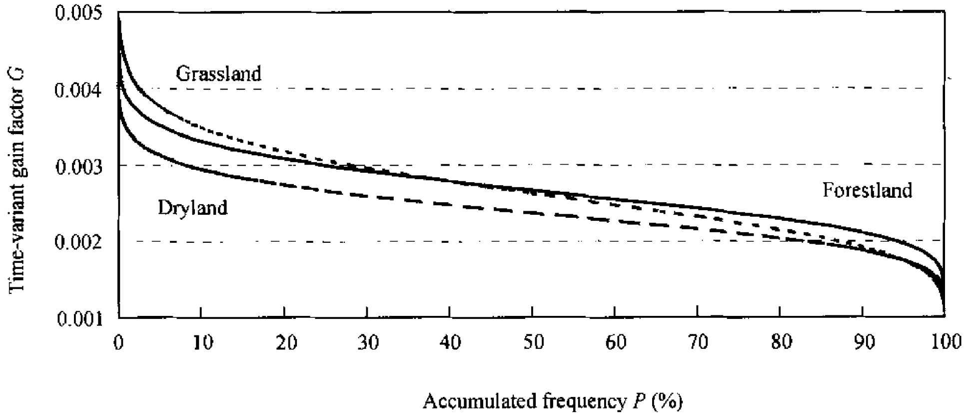

Fig. 4. A stochastic relation between different land covers and hydrological gain factor G in the Chaobai River basin of North China.

generation amount of forest cover is less than that of grassland.

Because of the intuitionistic concept of time variant gain factor, G, i.e., variable runoff coefficient and simple model structure, it is easy for DTVGM to establish the connection of various land uses with infiltration characteristics by using remote sensing information, and then estimate the impact of LUCC on hydrological cycle directly or indirectly. Thus, DTVGM could be a more workable approach to describe the time-space variation of water cycle and quantify the relationship between LUCC and hydrological response in the future.

3 Conclusions

(1) Nonlinearity of hydrological systems is one of the most difficult challenges in hydrological science. There are two different mathematical modeling approaches to describe the nonlinear system. One is the nonlinear Volterra functional series within a macro-scale, and the other is the nonlinear differential equation within a micro-scale. This research demonstrates that these two kinds of approaches could be transformed into each other under some conditions.

(2) Though the nonlinear approaches are more complex than the linear ones, it is possible to solve complicated hydrological problems using some simple methods. By establishing the connection between the time-variant gain factor G and soil moisture, a simple TVGM model can be developed with fewer parameters and higher precision. The research of TVGM inspires us to reveal rules from hydrological phenomena themselves and find direct relations out of complicated phenomena, despite of the traditional macro and micro approaches. This thought would be a new research aspect in terms of nonlinear hydrology, being worthy of further exploration.

(3) With respect to watershed hydrology in changing environment, the nonlinearity and uncertainty are obvious. Based on the nonlinear analysis, a distributed hydrological model (DTVGM) coupled with DEM has been developed. The developed method has been successfully applied to two river basins in China. DTVGM has been initially used to study the impact of LUCC on runoff generation processes. The integration of the nonlinear approaches within the distributed hydrological model would be a desired approach for hydrological simulation.

(4) The developed lumped and distributed time-variant gain models could be further extended by integrating groundwater simulation within its framework.

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